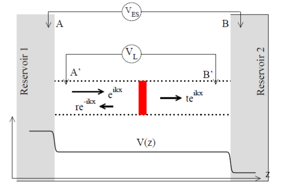
**Q1D Scattering**

Now let’s look at this in a Q1D framework. General schematic setup looks like this:



The Hamiltonian is given below.



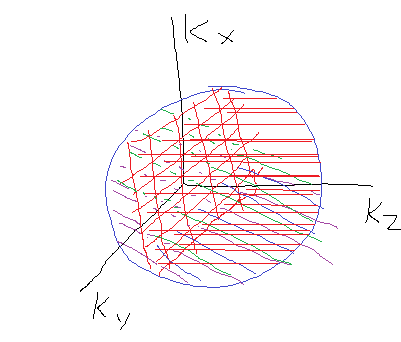
We can project this onto the transverse basis, assuming clamped boundary conditions for simplicity. Each transverse basis function we can think of as a ‘channel’. Then the building blocks of our solution in the potential-free region will be:



where A = LxLy, and W stands for the transverse variables x and y. Now kꓕ**n** can take on an infinitude of values, and kn is constrained by the energy equality above. The kꓕn which produce real kn are called open channels, and those which produce imaginary kn are called closed. The number of open channels is given below. The rest will be closed. It is possible that none are open. So *asymptotically*, our wavefunction in the leads will look like the following, but there may still be closed channel tail contributions extending past the region of the potential, into the leads.



The # of open channels is equal to the # of kperp states with kz values that lie on the Fermi-surface of constant energy. This is represented by the diagram:



and given by the inequality:



which leads to:



The next question is, ‘how do we connect them?’ First put the entire Schrodinger equation in the transverse basis.



Then we’ll recognize that:



and so have:



If Vnm(z) isn’t diagonal, then we’d have to diagonalize the potential operator. Let it be diagonalized by matrix U such that (implicit summation over repeated indices)



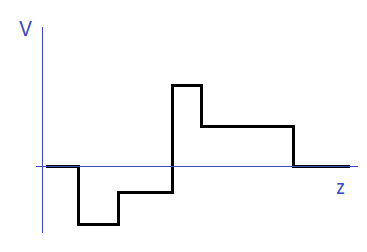
Then we have:



ψ carries the longitudinal momentum, roughly kα= √2m[E – α(z) - kꓕ2/2m], which can be imaginary. And so our solution would be:



where we use the WKB approximation to for sake of discussion. Let’s say we have some potential V(x,y,z) within region (z1, z2).



Then our solution looks like this. We’d presume the Am+ all known for open channels (and must be 0 for closed ones). Then we’d have to solve for all the other coefficients. Imposing continuity, differentiability conditions at z1,2 would suffice as it gives four sets of ∞ equations, matching four sets of ∞ unknowns.



Say V(x,y,z) = V(z) = piece-wise constant. Then in each piece, we’ll just have U diagonal, and the φ(z) will be the running waves, or decaying exponentials, left/right. Note the longitudinal kn, associated with a given kꓕn, will be different from region to region, and if E < V, they will even be all imaginary (closed). Also observe that the # of open channels in a region will vary inversely to V, and can also be zero, if E < V in that region.



where,



Now let’s consider the current. In each lead, it would be as follows. Closed channels do not contribute any current in the asymptotic regions, but they may in the interstitials, and in fact must, for current conservation. Basically if the we have both +kx, and –kx components in the interstitial, then the evanescent mode will carry current.



And then we can define transmission/reflection coefficients in terms of these:

